COMPASSS Working Paper 2010-61

www.compasss.org

Set-Relational Fit and the Formulation of Transformational Rules in fsQCA

Alrik Thiem

Center for Comparative and International Studies (CIS)
ETH Zurich and University of Zurich

Center for Security Studies (CSS)
ETH Zurich

ABSTRACT

Interest in the application of fuzzy set Qualitative Comparative Analysis (fsQCA) has increased markedly among political scientists in recent years. Although fsQCA is often contrasted with regression analysis, questions of functional form have thus far been hardly addressed. However, functional form matters as much in the former tool kit of social enquiry as it does in the latter. This article demonstrates how coverage as a measure of set-relational fit can guide the formulation of transformational rules similar to the use of the coefficient of determination in regression-analytic tests of functional misspecification. Interaction effects between membership function and crossover anchor choice on coverage are analysed in the context of set-theoretic relations of sufficiency between condition and outcome. Depending on the relative location of the crossover anchor, changes in the transformational rules by which base variables are calibrated can increase or decrease coverage similar to the effect of changes in functional specification on the coefficient of determination. Most importantly, significant reductions in uncovered membership relative to total membership provide an explicit and transparent foundation on which calibration strategies can be based.

Introduction[1]

Interest in fuzzy set Qualitative Comparative Analysis (fsQCA) has increased markedly among political scientist in recent years, with applications in fields ranging from constitutional executive control (Pennings 2003), over government backing for supra-nationalism in the EU (Koenig-Archibugi 2004), support for far-right parties (Veugelers and Magnan 2005), post-communist respect for civil liberty (Skaaning 2007), social inequality in education (Freitag and Schlicht 2009) and unpopular social policy reforms (Vis 2009), to effects of direct democracy on minority rights legislation (Christmann 2010).[2] The idea of fuzzy sets had already been introduced by Zadeh (1965) as a modelling language suited to the multivalence inherent in the soft sciences, but while it has found much favour with the information science and engineering community, the popularization in political science had to await the breakthrough contributions made by Ragin (2000, 2008).[3] As an alternative to traditional quantitative methods of scientific enquiry, fsQCA has established itself firmly in comparative research with small and medium-*N* data sets (Rihoux and Ragin 2009), but has also been used in studies with larger numbers of observations in the hundreds (e.g., Thiem 2010) and even thousands (e.g., Ragin and Fiss 2008). Thanks to the introduction of appropriate software, computational costs have been lowered tremendously (Ragin, Drass, and Davey 2006). Although fsQCA has undoubtedly enriched social scientists' methodological tool kit by emphasizing advantages over and differences from traditional regression analysis, aspects that bridge the two research methods have been largely neglected by its advocates.[4]

Specification tests of functional form on the basis of changes in the ratio of explained variation to total variation - the coefficient of determination R^2 - are part and parcel of regression analysis.[5] If the functional form between regressors and regressand is misspecified, the zero-conditional-mean assumption will be violated and estimators be biased. Usually, the coefficient of determination is likely to be significantly smaller than in a properly specified

[1] I thank Gary Goertz, Emelie Lilliefeldt, Mario Quaranta, Charles Ragin, Matthew Ryan, Bianca Sarbu, Maarten Vink and the two anonymous reviewers for helpful comments and suggestions. A previous version of this paper was presented at the 3rd ECPR Graduate Conference, Dublin City University, 30 August – 1 September 2010.

[2] For a review of fuzzy set applications in political science, see Nurmi and Kacprzyk (2007).

[3] For one of the first popular books on fuzzy logic and a very good introduction to its evolution in the hard sciences, see Kosko (1993). The first scientific text on the use of fuzzy sets in the social sciences has been presented by Smithson (1987).

[4] Smithson and Verkuilen (2006) are a notable exception.

[5] For example, Ramsey's (1969) regression specification error test (RESET) checks for non-linear relationships by adding polynomials of the fitted values $\hat{y}$ to the model $y = \beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k + \delta_1 \hat{y}^2 + \delta_2 \hat{y}^3 + \delta_3 \hat{y}^4$. If the difference in explained variation is large enough, the *F*-statistic on δ will be significant. Similarly, non-nested alternatives can be tested by means of an encompassing hyper-model (Mizon and Richard 1986), or a model augmented with the fitted values from the alternative (Davidson and MacKinnon 1981).

model.[6] While none of the Gauss-Markov assumptions required for the unbiasedness of the estimators has an equivalent in fsQCA, coverage provides a simple measure of empirical importance similar to the coefficient of determination in regression analysis (cf. Ragin 2008: 63).[7] As such, coverage determines the relevance that is attached to a causal path. However, while much textbook space has been expended on the specification of anchors, a discussion about the direct and indirect effects of using different membership functions has not been initiated so far. In point of fact, it has even been discouraged.

It is argued in this article that, for applications where fuzzy set membership scores are assigned by transforming interval or ratio-scale base variables, differences in coverage can be a useful criterion in the decision on a specific functional form in as much the same way as statistically significant differences in the coefficient of determination suggest different functional relations. This parallel notwithstanding, in fsQCA the question of functional form does not directly arise in the evaluation of the set-relational fit, but already during the process of calibration. Under some conditions, the choice of different membership functions induces differences in coverage, under others it does not. The relative location of the crossover anchor is the decisive interaction "variable" in this respect. The main conclusion to be drawn from the results of this article can be summarized as follows: By invoking differences in coverage, the choice for a specific membership function receives an explicit and transparent foundation in strategies aimed at generalizations as well as case-centred comparative research. The way in which base variables are transformed thus in turn determines the relevance scholars attach to causal paths.[8]

Four membership functions that act as parsimonious baseline choices for transformations to fuzzy sets with continuum endpoint labels regularly used in political science, such as "democratic", "wealthy" or "social inequality", illustrate the case. More specifically, the logistic function - the standard membership function implemented in the fsQCA software (Ragin 2008; Ragin, Drass, and Davey 2006) - is compared to three alternatives. These are the linear function, the quadratic function, and the root function.[9] All are applied to transform simulated base variables into conditions whose set-theoretic relation to the outcome is consistent with sufficiency. In order to demonstrate the interactive effect

[6] I do not engage herein in discussions on the general or model-specific usefulness of this measure. See the controversy between King (1990), Lewis-Beck and Skalaban (1990), and Achen (1990).
[7] Coverage can be divided into raw coverage and unique coverage similar to the distinction between the multiple and the partial coefficient of determination, but this distinction is not necessary here, since minimal formulas consist of a single condition as the only prime implicant, essentially corresponding to a simple regression model with only one regressor.
[8] Consistent with standard terminology as suggested by Rihoux and Ragin (2009: 181-184), I will refer to the causal condition / condition variable simply as "condition" and to the outcome variable as "outcome".
[9] The term "root" will be used as shorthand to mean "square root".

between the membership function and the crossover anchor choice on the coverage scores of these relations, the analysis is conducted in a first step for three representative anchors and a specific base variable. In a second step, it is extended to a continuous range of anchors from a very large number of simulated base variables. The exclusion and the inclusion anchors are irrelevant in this connection, because they do not interact with the membership functions in the determination of coverage over the crucial sub-domains.[10]

The article is structured around three sections. Firstly, the concept of coverage is elaborated on in the context of set-theoretic relations indicating sufficiency. Although this article focuses on sufficiency, the logic behind the results is equally valid in the context of necessity relations. Secondly, the specific mechanism by which the values of a base variable are transformed into fuzzy set membership scores is explained in more detail. In the third section, the membership functions are brought together with the crossover anchor in order to demonstrate their interactive effect on coverage. Thus, the sections are ordered in the reverse way of the procedural protocol usually adhered to in fsQCA applications, because the downstream effects of the initial calibration procedure have so far been least understood. The conclusions recapitulate the argument, point towards avenues for future research and suggest ways to advance and improve scholarly work.

Coverage in Set Relations of Sufficiency

Set-theoretic relations between fuzzy sets can be conveniently visualized using a bivariate plot as shown in Figure 1. It displays the set membership score of the condition **C** on the abscissa, and the set membership score of the outcome **O** on the ordinate. Cases above or on the diagonal line **O** = **C** are consistent with a set-theoretic relation of sufficiency between **C** and **O**, concisely expressed as $\mathbf{C} \subseteq \mathbf{O}$. In contrast, those below or on the diagonal are consistent with a set-theoretic relation of necessity between **C** and **O**, expressed as $\mathbf{C} \supseteq \mathbf{O}$. For example, *q* in Figure 1 is consistent with $\mathbf{C} \subseteq \mathbf{O}$, but inconsistent with $\mathbf{C} \supseteq \mathbf{O}$, whereas *p* is inconsistent with $\mathbf{C} \subseteq \mathbf{O}$, but consistent with $\mathbf{C} \supseteq \mathbf{O}$. If all cases fall above the diagonal, the set-theoretic relation is perfectly consistent with hypotheses about causal relations of sufficiency. If they all fall below it, it is perfectly consistent with hypotheses about causal

[10] This is not exactly true of the logistic function, which is defined over only two, instead of four, sub-domains, so that cases below the exclusion anchor always contribute more to coverage than the same cases transformed by the other membership functions. However, cases above the inclusion anchor always contribute less to coverage, so that any resulting differences are highly likely to be negligible.

relations of necessity.[11] If all cases fall on the diagonal, the set-theoretic relation indicates necessity and sufficiency. In the remainder of this article, the focus will be on set-theoretic relations of sufficiency.[12]

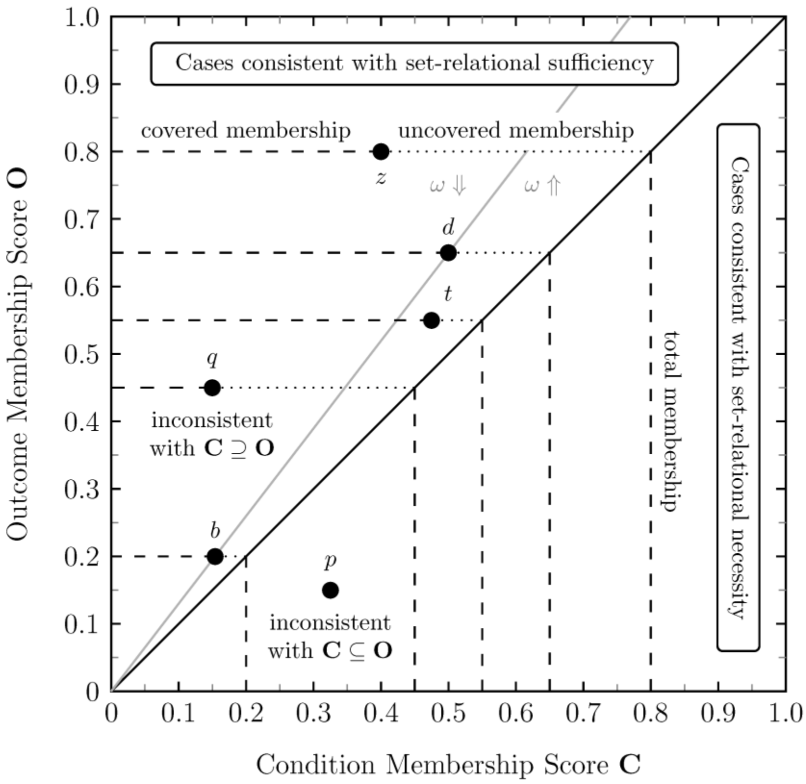

Figure 1: Coverage and Fuzzy Set-Theoretic Relations

Once sufficiency of a condition for an outcome has been established based on an appropriate proportion of cases that are consistent with this set-theoretic relationship, the measure of coverage provides a simple way to assess its empirical relevance by means of the degree to which set membership in **C** covers set membership in **O** (cf. Ragin 2006: 292). Often consistency works against coverage, but any assessment of the latter requires that the former has already been adequately addressed by the researcher.[13] If coverage is low, the condition may be trivial in its relevance to establish sufficiency. Conversely, high coverage points

[11] Set-theoretic relationships need not necessarily assume causality. However, causal relationships are of most interest to social scientists.
[12] This choice is without loss of generality with regards to the logic behind the results, since assessments of coverage in relations of necessity simply reverse the constituent parts of their formulas.
[13] Different measures of consistency, also sometimes referred to as inclusion indexes, exist, but the standard index is the fraction of membership in the condition that is covered by membership in the outcome.

towards a non-trivial condition.[14] Let coverage be denoted by ω. In set-theoretic relations of sufficiency it is computed by dividing the scalar cardinality of the intersection between **C** and **O** by the scalar cardinality of **O**.[15] Put differently, it is the fraction of total membership in the outcome that is covered by membership in the condition with

$$\omega(\mathbf{C} \subseteq \mathbf{O}) = \frac{\sum_{i=1}^{n} \min(c_i, o_i)}{\sum_{i=1}^{n} o_i}.$$

As such, coverage closely resembles the coefficient of determination R^2 in regression analysis, which is the fraction of the sample variation in the regressand that is explained by variation in the regressors

$$R^2 = \frac{\sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2}$$

Both measures provide a summary statistic of how closely the model fits the data. The construction of coverage implies three simple, yet important facts. Firstly, given the same condition membership score $c_i = c_j$ with $i \neq j$, any case i with a lower outcome membership score o_i than o_j always contributes more to ω. Secondly, given the same outcome membership score $o_i = o_j$, any case i with a higher condition membership score c_i than c_j always contributes more to ω. And thirdly, given the same ratio of covered membership to total membership between two cases i and j, it is irrelevant to which degree they are members of **C** and **O** as each contributes equally to ω.[16] Graphically, this would include any case on the grey isoquant through *d* shown in Figure 1, excluding the origin. Case *b* contributes exactly as much to ω as *d*, whereas *t* contributes more, and *z* contributes less.

Thus, coverage provides a simple measure of set-relational fit that assesses empirical importance similar to the coefficient of determination, which changes with the ratio of explained variation to total variation as a result of, for example, a change in the functional relationship between regressors and regressand. This relationship does not directly arise between the condition and the outcome, but during the process of designing the fuzzy set over the entire domain as given by the base variable. Referred to as calibration, that process should be implemented with a view to ensuring a maximum degree of correspondence between the values of the base variable and the qualitative meaning attached to the numerical values of the

[14] Empirical triviality may or may not coincide with theoretical triviality, depending on factors in the respective research context, such as previous evidence.
[15] The scalar cardinality of a fuzzy set is the sum of membership degrees of its cases.
[16] Essentially, this third point is a generalization of the preceding two.

fuzzy set.[17] However, very few base variables have a natural coding in the unit interval, and even if that is the case, pure correspondence in theoretical range does not guarantee qualitative correspondence. The mechanism that establishes an explicit rule for transforming base variable values to corresponding fuzzy set scores is the membership function.

Calibration and Membership Functions

As Ragin (2009: 93) puts it, "[...] the calibration of fuzzy sets is a key operation, to be performed with great care". Principally, calibration can take three different routes. Direct methods draw on expert knowledge to provide membership scores. Indirect methods also use expert knowledge, but this has to be processed, often via some curve-fitting method, in order to translate it into fuzzy set membership scores. Another method frequently used in political science applications is referred to as "assignment by transformation" (Verkuilen 2005).[18] It is based on some function that transforms existent data on an interval or ratio-scaled base variable, such as GDP, power capabilities or public opinion, into fuzzy set membership scores while taking the substantive meaning of the values on their underlying scale with regards to the label of the fuzzy set into account. This correspondence between numerical values and qualitative meaning distinguishes the fuzzy set from its base variable, which only measures *degree*, but not *kind*.

Two main approaches to transformational assignments can be distinguished to ensure correspondence between numerical values and qualitative meaning (cf. Verkuilen 2005: 481f.). Data-based transformations are internally anchored. They often make use of measures of central tendency or spread characterizing the base variable, either because no substantive meaning of the fuzzy set label exists or it can be changed almost arbitrarily. Theoretical transformations, in contrast, are externally anchored. They depend on the substantive meaning of the values in the base variable with respect to the fuzzy set label.[19]

Irrespective of whether existing knowledge suggests that it be data-based or theoretical, fuzzy sets to be designed by the method of transformational assignment first require the choice of anchors from their base variable which are mapped onto their corresponding cut-offs in the fuzzy set. Let fuzzy sets be denoted by $\mathbf{A}$, base variables by X and their specific values by x_i with $i = 1, 2, \ldots, n$, anchors from X by τ, and cut-offs in $\mathbf{A}$ by α. Three cut-offs

---

[17] Calibration is also often referred to as fuzzification (cf. Clark et al. 2008: 44).

[18] The indirect method listed here corresponds to the one suggested in Ragin (2008), but his direct method is the same as the method of assignment by transformation.

[19] Data-based transformations are not recommended by most advocates of QCA (e.g., Ragin, Strand, and Rubinson 2008: 16f.).

are relevant, namely α_0 for exclusion from **A**, $\alpha_{0.5}$ for the crossover at which membership in **A** is most ambiguous, and α_1 for inclusion in **A**. Accordingly, three anchors are required. The exclusion anchor τ_e is mapped onto the exclusion cut-off α_0, the crossover anchor τ_c onto the crossover cut-off $\alpha_{0.5}$, and the inclusion anchor τ_i onto the inclusion cut-off α_1.[20]

For example, the Regional Authority Index (X) developed by Marks, Hooghe and Schakel (2008) measures the extent to which the government of a country is structurally layered. It has no theoretical meaning that is external to its construction because specific values (x_i) are only qualitatively interpretable against their distribution. There is no external guidance as to whether a country is more in than out of the set of countries with a highly decentralized government structure (**A**). Measures of central tendency or spread could thus guide the choice of τ_e, τ_c and τ_i to anchor α_0, $\alpha_{0.5}$ and α_1.

In contrast, national public opinion figures taken to measure net support for a particular government policy (X) require an external specification of τ. In order to design the set of countries with a highly supportive citizenry regarding that particular policy (**A**), a net support rate of $\tau_c = 0$ provides a natural way of anchoring $\alpha_{0.5}$. If $x_i < 0$, there will be more opponents than proponents, and the country should be more out of than in **A**. Conversely, if $x_i > 0$, there will be fewer opponents than proponents, and the country should be more in than out of **A**. Most probably, there may exist more disagreement about the criteria used in anchoring α_0 and α_1, but it is clear that they should not be data-based, either.

Once τ has been specified, the first step of the calibration procedure is completed. Before X can then be transformed into **A** in the second step, an explicit transformation rule is required: the membership function. As many political science concepts are represented as continuum endpoint concepts, functions that are strictly increasing over some sub-domains, and possibly at least non-decreasing over others, are obvious candidates. In particular, cumulative distribution functions have the desired properties. However, in contrast to many studies which use regression analysis and argue for or against particular functional relationships in their models, researchers using fsQCA have so far been silent on their motivation to establish particular transformational rules. Often advice given in textbooks is not very helpful: "In practice, fuzzy set theory works by drawing a curve between opposites [...]. Information, substantive knowledge and theories help us to draw this curve. FS [*fuzzy sets*] can take all possible shapes (linear, S-function, Bell-curve, etc), where the specific shape should resemble the given concept as closely as possible." (Kvist 2006: 174). But what type

[20] I will refer to the "crossover cut-off" simply as "crossover".

of curve does the set of, for instance, "countries with high power capabilities" suggest? Linear, quadratic or logistic? Schneider and Wagemann (2007) do not even mention the issue of membership functions except in a short paragraph reference to Ragin (2008).

In the standard text on fsQCA, Ragin (2000: 171) himself argues that "it is important not to focus on mathematical functions". Similarly, in the sequel he notes that calibration procedures seem so forbidding that "the complex set of computational steps [...] can be accomplished with a simple compute command using the software package fsQCA" (Ragin 2008: 91). This does not quite seem the way to go forward in promoting methodological literacy. First-year students in mathematics are advised not to trust results of which it is not known how exactly they have come about, be they produced by software or not. Without opening the black box of calibration to better understand its direct as well as indirect effects, users of fsQCA are highly likely to remain as "push-button" as Berg-Schlosser et al. (2009: 14) accuse the vast majority of scholars using regression analysis to be. The membership function, as Verkuilen (2005: 464f.) puts it, "is the fundamental quantity necessary to use fuzzy sets", but without an understanding of the fundamentals the simple push of a button may have consequential downstream effects to which researchers will remain oblivious. Membership functions are as important as anchors, and it is in fact pointless to treat them as separate elements in the calibration process, or as steps in the procedural protocol which do not require equal care.

Consider the procedure applied by Koenig-Archibugi (2004) to assign condition membership scores to EU member states in the set of countries with high public support for supranational integration in relation to the Common Foreign and Security Policy.[21] For simplicity, let this set be denoted by **S**. The transformational rule the author establishes is a simple cumulative distribution of a uniform density function that normalizes the set and which is only mentioned in a footnote. Let $\mu_{\mathbf{S}}$ denote that function, let x_i be the i^{th} value from the base variable X measuring net public support, let τ_e equal the minimum value in X and let τ_i equal the maximum value in X. Then the membership function

$$\mu_{\mathbf{S}_i} = \frac{x_i - \tau_e}{\tau_i - \tau_e}$$

looks like an appropriate transformation. It implies a constant rate of convergence towards set inclusion between its endpoints and is strictly increasing over its entire domain. Nonetheless, by choosing a function which only accommodates τ_e and τ_i, Koenig-Archibugi neglects the

[21] The author uses general identity scores from Eurobarometer Survey figures to proxy support.

qualitative meaning of the now mechanistically-defined crossover anchor $\tau_c = (\tau_i + \tau_e)/2$. Substituting the values of the two most extreme countries, namely Luxembourg and the UK, into the formula yields $\tau_c = (49.1+(-15.2))/2 = 16.95$. With $\tau_c = 16.95$, even countries whose net support rates are as solid as 17 per cent only have ambiguous membership in **S**. Germany, where public backing for European integration has had a broad base, does not even pass the crossover with a set membership score of 0.47. To make out a strong case for this assignment seems but impossible. The vast majority of political scientists familiar with German politics would consider its population, masses and elites alike, comparatively supportive of European integration and supra-nationalism (e.g., Banchoff 1999; Marcussen et al. 1999). The membership function used by Koenig-Archibugi induces a not inconsiderable degree of incongruity into the correspondence between *X* and **S**.

This single example illustrates just how important it is to consciously control the calibration process, in particular the anchoring of $\alpha_{0.5}$ in the membership function, for which the investigator should always present a rationale (Ragin 2009: 92). It is the reference point of maximum ambiguity above which cases are considered more in than out of the set, and below which cases are considered more out of than in the set. Thus, the lower τ_c relative to τ_e and τ_i, the more cases will shift towards stronger set membership, and the higher τ_c, the more cases will shift towards weaker membership. As a result, the anchoring of $\alpha_{0.5}$ should always be guided by internal or external criteria, and it should always precede the formulation of transformational rules.

In the example above, the absence of τ_c from the membership function left the same underspecified. Setting $\tau_c = 0$ - the point where there are as many opponents as proponents of supra-nationalism - and incorporating it into μ_S would have been a suitable adjustment. A split of the function at τ_c would have turned it into a piecewise rule that had better ensured correspondence between *degree* and *kind*. In fact, the possibility of an independent choice of τ_c requires a membership function to be piecewise-defined. Essentially, the number of functions that fulfil these criteria is infinite, and the exact formulation of transitional rules is most often a judgement call unless clear contextual evidence of how the concept in question is used exists. However, simplicity and easy constructability on the basis of little information are important ancillary criteria in this process (Bojadziev and Bojadziev 2007: 23; Klir, St. Clair, and Yuan 1997: 80).

Besides the linear membership function used by Koenig-Archibugi (2004), by far the most popular choice for the assignment of fuzzy set membership scores by transformation has been the logistic function.[22] It is the default used by the current version of the fsQCA software. Very similar in shape is the quadratic function. Both are marked by increasing rates of convergence towards set inclusion between τ_e and τ_c, referred to as concentration, and decreasing rates between τ_c and τ_i, called dilation.[23] In contrast, the root function is characterized by dilation between τ_e and τ_c, and concentration between τ_c and τ_i. It is the mirror image of the quadratic function. All four alternatives represent common baseline choices in the assignment of fuzzy set membership scores. There is no *ex ante* reason for why the logistic function is preferable to the linear or root function unless clear context-bound evidence of concentration below τ_c and dilation above it exists. Even more so, very similar patterns of concentration and dilation render the choice of the logistic in preference to the

Table 1: Four Baseline Membership Functions

Function	Formula	Domain
Linear	μ_{lin_i}	$= \begin{cases} 0 & \text{if } x_{min} \leq x_i \leq \tau_e, \\ \frac{1}{2}\left(\frac{\tau_e - x_i}{\tau_e - \tau_c}\right) & \text{if } \tau_e < x_i \leq \tau_c, \\ 1 - \frac{1}{2}\left(\frac{\tau_i - x_i}{\tau_i - \tau_c}\right) & \text{if } \tau_c < x_i \leq \tau_i, \\ 1 & \text{if } \tau_i < x_i \leq x_{max}. \end{cases}$
Quadratic	μ_{quad_i}	$= \begin{cases} 0 & \text{if } x_{min} \leq x_i \leq \tau_e, \\ \frac{1}{2}\left(\frac{\tau_e - x_i}{\tau_e - \tau_c}\right)^2 & \text{if } \tau_e < x_i \leq \tau_c, \\ 1 - \frac{1}{2}\left(\frac{\tau_i - x_i}{\tau_i - \tau_c}\right)^2 & \text{if } \tau_c < x_i \leq \tau_i, \\ 1 & \text{if } \tau_i < x_i \leq x_{max}. \end{cases}$
Root	μ_{root_i}	$= \begin{cases} 0 & \text{if } x_{min} \leq x_i \leq \tau_e, \\ \frac{1}{2}\left(\frac{\tau_e - x_i}{\tau_e - \tau_c}\right)^{\frac{1}{2}} & \text{if } \tau_e < x_i \leq \tau_c, \\ 1 - \frac{1}{2}\left(\frac{\tau_i - x_i}{\tau_i - \tau_c}\right)^{\frac{1}{2}} & \text{if } \tau_c < x_i \leq \tau_i, \\ 1 & \text{if } \tau_i < x_i \leq x_{max}. \end{cases}$
Logistic	μ_{log_i}	$= \begin{cases} \left(1 + \mathrm{e}^{-\left[(x_i - \tau_c)\left(\frac{-\log(19)}{\tau_e - \tau_c}\right)\right]}\right)^{-1} & \text{if } x_i < \tau_c, \\ \left(1 + \mathrm{e}^{-\left[(x_i - \tau_c)\left(\frac{\log(19)}{\tau_i - \tau_c}\right)\right]}\right)^{-1} & \text{if } x_i \geq \tau_c. \end{cases}$

[a] x_i; x_{min}; x_{max}: base variable value; minimum base variable value; maximum base variable value
τ_e; τ_c; τ_i: anchor for exclusion cut-off α_0; anchor for crossover $\alpha_{0.5}$; anchor for inclusion cut-off α_1

[22] While Ragin (2000: 171) still leaves the membership function choice to the judgement of the researcher, only the logistic function is proposed by Ragin (2008).
[23] In fact, the linear function is a special case of the polynomial function class to which also the quadratic belongs. But it is unique in its neutrality with respect to concentration and dilation.

quadratic function a question of computational convenience rather than substantive considerations.

While the three alternatives are capable of producing set membership scores in the unit interval, practically the logistic function is not. Thus, I follow Ragin (2008) with regards to the logistic function in using $\alpha_{0.05}$ instead of α_0 as the exclusion cut-off, and $\alpha_{0.95}$ instead of α_1 as the inclusion cut-off. Table 1 lists the piecewise-defined linear function μ_{lin}, the quadratic function μ_{quad}, the root function μ_{root} and the logistic function μ_{log}. All but the logistic function are defined over four sub-domains, the first of which contains all cases whose value falls below or equals τ_e. Over this sub-domain, set membership scores are always zero. The second sub-domain contains all cases with values between τ_e and τ_c, including τ_c. Over this range, cases have only weak set membership. The third sub-domain contains all cases whose value lies between τ_c and τ_i, including τ_i. Their associated set membership can be described as strong. All remaining cases whose value falls above τ_i are full members of the set. With regards to the logistic function, only two sub-domains are needed, the first for all cases with values below τ_c, and the second for all cases with values equal to or above τ_c.

The individual effect of changes in the crossover anchor and the membership function on the distribution of membership scores in **A** can be seen in Figure 2. An artificial base variable X with random values $x_1, x_2, \ldots, x_{30}$, representing a medium-N data set, was generated from a standard normal distribution and positivized by adding the modulus of the minimum to each value.[24] The exclusion and inclusion anchors τ_e and τ_i were fixed at the 5th and 95th percentile of X respectively. They are thus an example of anchors whose specification is based on internal criteria since the base variable is generic without a qualitatively meaningful scale. Each of the four membership functions, μ_{lin} in sub figure 2a, μ_{quad} in 2b, μ_{root} in 2c and μ_{log} in 2d was then combined with three different crossover anchors τ_c in designing **A**.

The first of these, $\tau_l = \tau_e + (\tau_i - \tau_e)/4$ (light-grey dots), is a relatively low value between τ_e and τ_i, causing the vast majority of cases to have strong membership in **A**. In the second specification, $\tau_m = \tau_e + (\tau_i - \tau_e)/2$ (grey dots), about as many cases lie above $\alpha_{0.5}$ as below it. The third anchor, $\tau_h = \tau_e + 3/4(\tau_i - \tau_e)$ (black dots), models a situation with a relatively high threshold for strong membership so that the vast majority of cases have only weak membership in **A**. All functions are symmetric around τ_c when $\tau_c = \tau_m$. As τ_c decreases to τ_l, more and more cases are "elevated" to a score above $\alpha_{0.5}$, while as it increases to τ_h, more and more cases are "relegated" to a score below $\alpha_{0.5}$.

[24] This was simply done for reasons of graphical presentation. R 2.11.1 was used in producing the results in the remainder of this article. All code is available from the author on request.

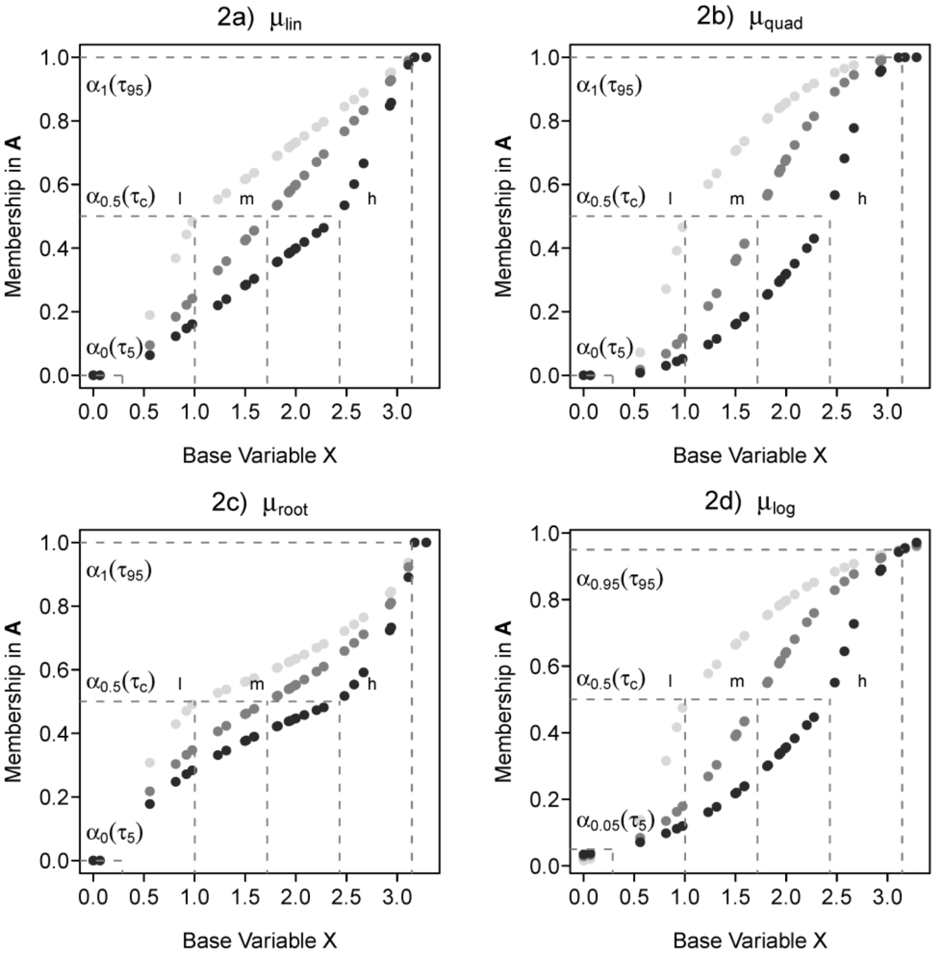

Figure 2: Calibration with varying membership function and crossover anchors

After the main concepts of coverage, crossover anchor and membership function have been introduced and their relationship to each other explained, the next section now shows how the two steps of calibration, namely specification of τ and μ, impact on coverage. More specifically, it shows why coverage depends on the interaction between functional form and the relative location of τ_c, providing a suitable rationale for choosing one membership function over another if this criterion is consistent with the researcher's calibration strategy. It is further demonstrated that the logistic function may not be a good choice in this connection. In fact, it may never be the best choice.

Coverage and the Choice of Membership Function

Recall that coverage is the ratio between covered membership and total membership, similar to the coefficient of determination being the ratio between explained and total variation. This way of computation implies that coverage always increases with decreasing total membership holding covered membership fixed, increasing covered membership holding total membership fixed, or any other combination that fulfils the following inequality

$$\frac{c_i}{o_i} < \frac{ac_i}{bo_i} \leq 1$$

with $0 < a \leq 1/c_i$ and $0 < a \leq 1/o_i$. In order to keep things simple, I focus on the case where total membership is fixed but covered membership is allowed to vary. This set-up essentially models a situation in which the outcome has already been designed but the condition not yet been calibrated.[25] Let **A** used for illustrating the calibration process in the preceding section and shown in Figure 2 be the condition **C**. Furthermore, let **O** be a generic outcome so that any single case i only differs on c_i across μ, but not on o_i, and let r be a random variable from a uniform distribution in the unit interval. Then o_i is determined by

$$o_i = \max\big(\mu(x_i)\big) + r\left(1 - \max\big(\mu(x_i)\big)\right).$$

Using this formula to construct **O** ensures that all set-theoretic relations of sufficiency are perfectly consistent because o_i is always at least as large as any $\mu(x_i) = c_i$ but never exceeds the set ceiling.[26] Figure 3 visualizes all twelve resulting relationships, in sub-figure 3a for the low crossover anchor τ_l, in 3b for the medium crossover anchor τ_m and in 3c for the high crossover anchor τ_h. The coverage scores resulting from the combination of each membership function with each crossover anchor are summarized in the bottom-right corner.

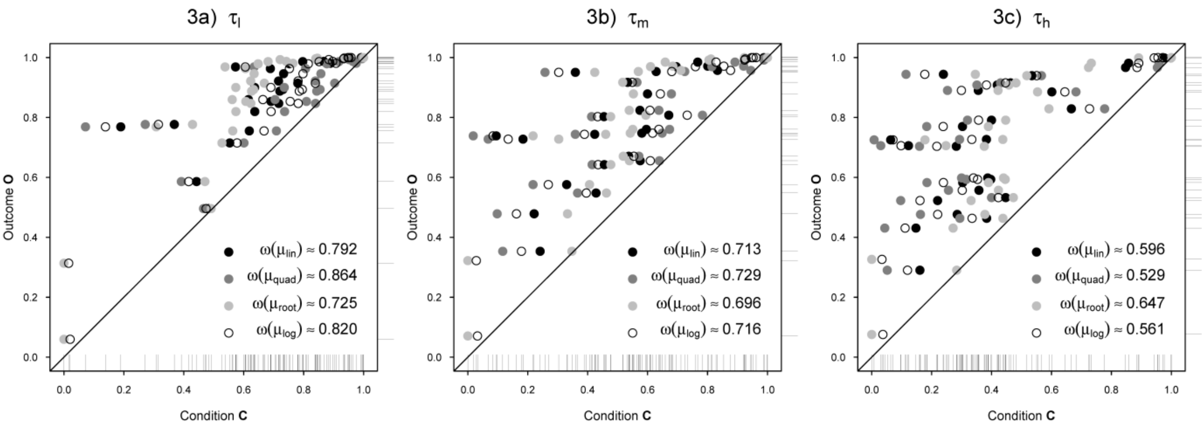

Figure 3: Coverage under different calibration scenarios

[25] The analogy in regression analysis would be the transformation of a regressor to increase the model fit, such as logging or squaring, while leaving the regressand untransformed.

[26] If coverage was allowed to work against consistency, the resulting loss in the latter would have to be weighed up against gains in the former, but this is another question that requires a separate treatment.

In the case of $\tau_c = \tau_l$, ω is highest at about 86% for the case when X is transformed by μ_{quad}, 82% by μ_{log}, 79% by μ_{lin} and 73% by μ_{root}. The difference amounts to a striking 13 percentage points (pp) between the "best-performing" membership function μ_{quad} and the "worst-performing" μ_{root}, to 7 pp when compared to μ_{lin} and to 4 pp in comparison with μ_{log}. When $\tau_c = \tau_m$ at the medium distance between the exclusion and the inclusion anchor, the performance ordering remains the same, but differences between ω shrink considerably while its absolute value decreases across all functions, but to varying degrees. In the case of $\tau_c = \tau_h$, ω keeps decreasing to 65% for the case when X is transformed by μ_{root}, 60% by μ_{lin}, 56% by μ_{log} and 53% by μ_{quad}, but differences increase again to a maximum of about 12 pp between μ_{root} and μ_{quad}, to 9 pp between μ_{root} and μ_{log}, and to 5 pp between μ_{root} and μ_{lin}. To summarize this first step, coverage depends considerably on the membership function by which X is transformed, but it does so in interaction with the location of the crossover anchor. For a relatively low value of τ_c, the quadratic function seems to be the transformational rule of choice, whereas for a relatively high value of τ_c, the root functions performs much better. At a medium value of τ_c no clear picture emerges.

Whether these differences in ω are representative of base variables that are roughly normally distributed cannot be inferred from a single simulation. To examine how ω behaves across a large set of base variables of equal size, the simulation of X is repeated a thousand times and calibration again carried out for each case of τ_c. Figure 4 graphs the distributions of ω and their means $\overline{\omega}$ from these simulations in kernel density plots, for the case of τ_l in 4a, τ_m in 4b and τ_h in 4c. The values of $\overline{\omega}$ listed in each sub-figure confirm earlier results from Figure 3 in that the ordering of performance in coverage remains the same for the cases of $\tau_c = \tau_l$ and $\tau_c = \tau_h$. In the case of $\tau_c = \tau_m$, however, any differences disappear almost completely, whereas variation in coverage remains largest for μ_{quad}, followed by μ_{log}, μ_{lin} and μ_{root}.

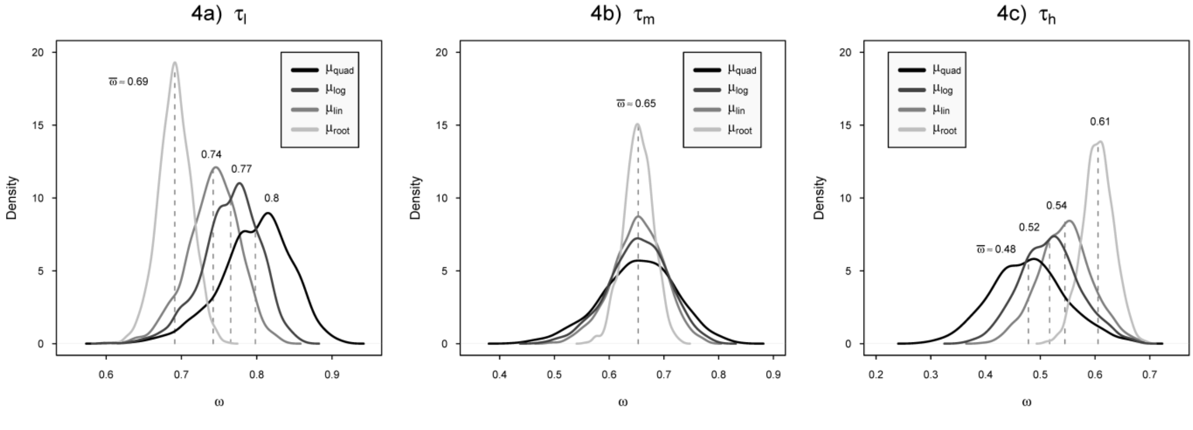

Figure 4: Distribution of coverage

When $\tau_c = \tau_l$, on average transformation by μ_{quad} instead of μ_{root} increases ω by about 11 pp, instead of μ_{lin} by about 6 pp and instead of μ_{log} by about 3 pp. Conversely, when $\tau_c = \tau_h$, transformation by μ_{quad} instead of μ_{root} decreases ω by about 13 pp, instead of μ_{lin} by about 6 pp and instead of μ_{log} by about 4 pp. Coverage cannot be changed on average by any transformation when $\tau_c = \tau_m$. Table 2 shows the results of *t*-tests for statistically significant differences in $\bar{\omega}$ when going from transformation through the membership function listed in the respective row of the table to transformation through the membership function listed in the respective column. All differences $\Delta\bar{\omega}$ are highly statistically significant at the 0.001 level except those generated when $\tau_c = \tau_m$.

Table 2: Welch t-tests of $\Delta\bar{\omega}$ [a]

μ		μ_{quad}	μ_{root}	μ_{log}
μ_{lin}	τ_l	0.0560*	-0.0510*	0.0233*
	τ_m	-0.0002	0.0001	-0.0001
	τ_h	-0.0666*	0.0605*	-0.0278*
μ_{quad}		τ_l	-0.1070*	-0.0327*
		τ_m	0.0003	0.0001
		τ_h	0.1271*	0.0388*
μ_{root}			τ_l	0.0744*
			τ_m	-0.0002
			τ_h	-0.0883*

[a] *: $p < 0.001$

So far, effects have only been analysed for three specifications of τ_c, but it would be interesting to see how coverage behaves across a continuous range of choices, spanning values from very close to the exclusion anchor to very close to the inclusion anchor. After all, the theoretically infinite number of ways to anchor $\alpha_{0.5}$ is only practically limited by the number of internal or external criteria that can be invoked defensibly in each specific case. Figure 5 shows how $\bar{\omega}$ varies across membership functions and with τ_c from a value close to τ_e at $\tau_c = \tau_e + k(\tau_i - \tau_e)$ with $k = 0.1$, to a value close to τ_i at $\tau_c = \tau_e + k(\tau_i - \tau_e)$ with $k = 0.9$, using the same data. Coverage scores are least sensitive to changes in τ_c when $\bar{\omega}_{root}$ is used for transformations, and most sensitive when $\bar{\omega}_{quad}$ is used. From the minimum value of τ_c to the medium value between τ_e and τ_i, $\bar{\omega}_{quad} > \bar{\omega}_{log} > \bar{\omega}_{lin} > \bar{\omega}_{root}$, while from the medium value between τ_e and τ_i to the maximum value of τ_c, $\bar{\omega}_{quad} < \bar{\omega}_{log} < \bar{\omega}_{lin} < \bar{\omega}_{root}$.

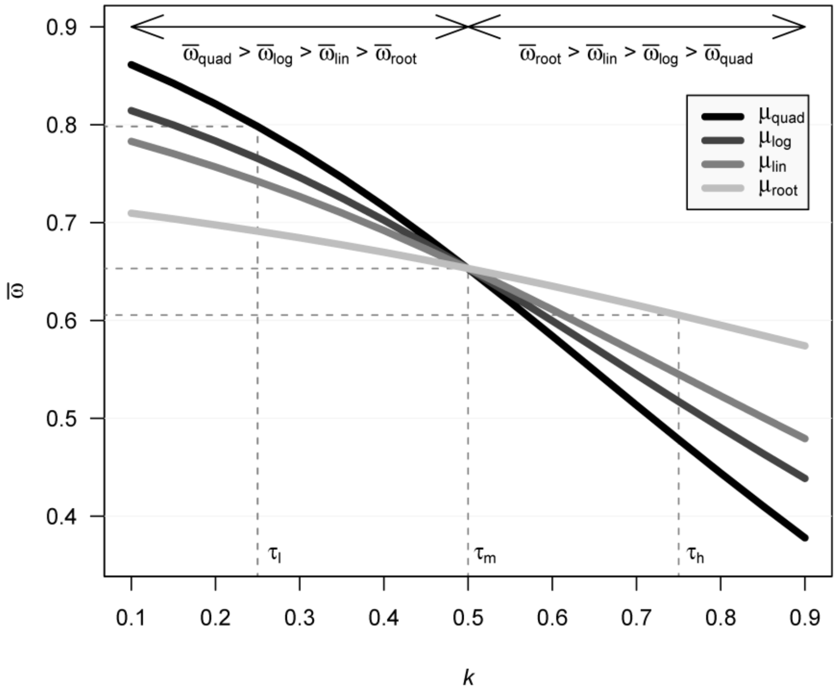

Figure 5: Average coverage scores over continuous crossover range

Does calibration through different membership functions increase or decrease coverage uniformly across all individual base variable values? In regression analysis, cases that are further away from the relation's mean-defined centre of gravity have potentially greater influence on the coefficient of determination. In fsQCA, no such centre exists. Values far away from the crossover anchor may have the same influence on coverage as values close to it. However, two membership functions that are not parallel between the exclusion and the inclusion anchor have at least one point at which the substantial effect on coverage produced by different transformational rules is greater than anywhere else along the range. Let $\Delta\omega(\mathbf{C} \subseteq \mathbf{O})$ be the total difference in ω between the initial transformation of X by some membership function μ_a and the alternative transformation by some other membership function μ_b so that

$$\Delta\omega(\mathbf{C} \subseteq \mathbf{O}) = \frac{\sum_{i=1}^{n} \min(\mu_b(x_i), o_i)}{\sum_{i=1}^{n} o_i} - \frac{\sum_{i=1}^{n} \min(\mu_a(x_i), o_i)}{\sum_{i=1}^{n} o_i}.$$

Then, $\Delta\omega(\mathbf{C} \subseteq \mathbf{O})/n$ would be the average additional contribution of each case induced by replacing μ_a with μ_b.[27] However, differences in ω generated by going from μ_a to μ_b are not equally attributable to each x_i, essentially making the distribution of all values $\tau_e \leq x_i \leq \tau_i$ the

[27] Another way of comparing the impact of two different membership functions μ_a and μ_b on condition set membership scores is a coincidence index, for example, $\frac{\sum_{i=1}^{n} \min(\mu_a(x_i), \mu_b(x_i))}{\sum_{i=1}^{n} \max(\mu_a(x_i), \mu_b(x_i))}$. I thank Charles Ragin for pointing this out. Generally, two membership functions for which differences in coverage are large would produce low scores on the coincidence index.

third factor in the determination of ω. This factor has been neglected in this article by making the realistic assumption that most data political scientists use in assignments by transformation is roughly normally distributed. More generally, let c_j be the condition membership score assigned to x_i by μ_b, and let c_i be the condition membership score assigned to all x_i by μ_a. Then the individual contribution $\Delta\omega_j(\mathbf{C} \subseteq \mathbf{O})$ of any case j to ω induced by a change from μ_a to μ_b is given by

$$\Delta\omega_j(\mathbf{C} \subseteq \mathbf{O}) = \frac{\sum_{i=1}^{n-j} \min(c_i, o_i) + \min(c_j, o_i) - \sum_{i=1}^{n} \min(c_i, o_i)}{\sum_{i=1}^{n} o_i}$$

where the first summation index in the numerator means "from $i = 1$ to n excluding j". Some cases j add to or subtract from ω substantially more than others, depending on their location relative to τ. Positive contributions to coverage are largest for those values x_i for which calibration produces the greatest positive difference between c_j and c_i, while negative contributions to coverage are largest for those values x_i for which calibration produces the greatest negative difference between c_j and c_i. There is a unique value x^{pos} for which positive contributions reach a maximum, and a unique value x^{neg} for which negative contributions reach a maximum. The values x^{pos} and x^{neg}, which may or may not correspond to some actual value $x_i \in X$, solve the following equation

$$\frac{\partial \mu_{\mathrm{a}}}{\partial x} = \frac{\partial \mu_{\mathrm{b}}}{\partial x}.$$

The absolute change $c_j - c_i$ evaluated at these particular values is then simply given by $\mu_b(x^{\text{pos}}) - \mu_a(x^{\text{pos}})$, $\mu_b(x^{\text{neg}}) - \mu_a(x^{\text{neg}})$ respectively. For example, the values $x = x^{\text{pos}}$ and $x = x^{\text{neg}}$ at which a change from μ_{quad} to μ_{root} generates the largest positive contribution to ω, the largest negative contribution respectively, both satisfy

$$\frac{\partial \mu_{\mathrm{quad}}}{\partial x} = \frac{\partial \mu_{\mathrm{root}}}{\partial x}.$$

Substituting the pieces of the functions over the sub-domain $\tau_e \leq x_i \leq \tau_c$ where $\mu_{\text{root}}(x) \geq \mu_{\text{quad}}(x)$ and taking their partial derivatives with respect to x yields

$$-\frac{1}{4(\tau_{\mathrm{e}} - \tau_{\mathrm{c}})\sqrt{\frac{\tau_{\mathrm{e}} - x}{\tau_{\mathrm{e}} - \tau_{\mathrm{c}}}}} = -\frac{\tau_{\mathrm{e}} - x}{(\tau_{\mathrm{e}} - \tau_{\mathrm{c}})^2}.$$

When this equation is now solved for x, $x = x^{\text{pos}}$ is given by

$$x^{\text{pos}} = \tau_{\text{e}} + \frac{\sqrt[3]{(-\tau_{\text{e}}^3 + 3\tau_{\text{e}}^2\tau_{\text{c}} - 3\tau_{\text{e}}\tau_{\text{c}}^2 + \tau_{\text{c}}^3)}}{2\sqrt[3]{2}}.$$

The closer any value x_i is to this point, and the more such values there are in X, the larger will be the increase in ω induced by a change from transformation of X through μ_{quad} to transformation through μ_{root}. Conversely, the point $x = x^{\text{neg}}$ at which a change from μ_{quad} to μ_{root} generates the largest decrease in ω is given by equating the partial derivatives with respect to x of those pieces of the functions over the sub-domain $\tau_{\text{c}} \leq x_i \leq \tau_{\text{i}}$ where $\mu_{\text{root}}(x) \leq \mu_{\text{quad}}(x)$ such that

$$\frac{1}{4(-\tau_{\text{c}} + \tau_{\text{i}})\sqrt{\frac{\tau_{\text{i}} - x}{-\tau_{\text{c}} + \tau_{\text{i}}}}} = \frac{\tau_{\text{i}} - x}{(-\tau_{\text{c}} + \tau_{\text{i}})^2}.$$

When this equation is now solved for x, $x = x^{\text{neg}}$ is given by

$$x^{\text{neg}} = \tau_{\text{i}} + \frac{\sqrt[3]{(\tau_{\text{c}}^3 - 3\tau_{\text{c}}^2\tau_{\text{i}} + 3\tau_{\text{c}}\tau_{\text{i}}^2 - \tau_{\text{i}}^3)}}{2\sqrt[3]{2}}.$$

The closer any value x_i is to this point, and the more such values there are in X, the larger will be the decrease in ω induced by a change from transformation of X through μ_{quad} to transformation through μ_{root}. It is now clear that when $\tau_{\text{c}} = \tau_{\text{m}}$, the average contribution to ω when the base variable is symmetrically distributed must be zero. The fraction in covered membership added to, respectively subtracted from, ω by all values $\tau_{\text{e}} \leq x_i < \tau_{\text{m}}$ is offset by all values $\tau_{\text{m}} < x_i \leq \tau_{\text{i}}$ so that on average

$$\Delta\omega(\mathbf{C} \subseteq \mathbf{O}) = \frac{\sum_{i=1}^{n} \min(\mu_{\text{quad}}(x_i), o_i)}{\sum_{i=1}^{n} o_i} - \frac{\sum_{i=1}^{n} \min(\mu_{\text{root}}(x_i), o_i)}{\sum_{i=1}^{n} o_i} = 0.$$

As τ_{c} shifts away from τ_{m}, this balance changes in accordance with the induced changes in the assignments of numerical set membership scores by μ_{a} and μ_{b}, making μ_{quad} the membership function of choice for all $\tau_{\text{c}} < \tau_{\text{m}}$ and μ_{root} for all $\tau_{\text{c}} > \tau_{\text{m}}$. Thus, if membership function choice is coverage-based, the logistic function is outperformed under virtually all conceivable crossover anchor configurations.

The results have important consequences, regardless of whether fsQCA is used in small-*N* studies with a comparative focus or large-*N* studies aimed at generalization. But it carries additional significance for case-oriented research. When τ_c approximates τ_m, coverage increasingly provides no guidance as to which membership function is preferred, but the exact relation between the condition **C** and the outcome **O** will still change, and considerably so for cases near x^{pos} and x^{neg}.

In summary, coverage can provide a very useful criterion in the formulation of transformational rules if that is consistent with the researcher's calibration strategy. On the basis of significant differences in coverage, the decision to use one specific membership function over the other so as to reduce uncovered membership relative to total membership receives an explicit and transparent foundation. This parallels the way scholars using regression analysis argue on the basis of tests for functional misspecification that evaluate reductions in unexplained variation relative to total variation for or against some specific functional form. Under very narrow circumstances coverage is of no guidance, but even if the focus is on case comparison rather than generalization, researchers still need to consider the effects of membership function choice, particularly if cases of interest lie close to points of maximum difference.

Conclusions

Coverage provides a measure of empirical relevance in the context of fuzzy set theoretic relations similar to the way the coefficient of determination is used to assess the empirical importance of regressors in explaining variation on the regressand. However, scholars applying fsQCA have made use of various membership functions in calibration strategies involving assignments by transformation without basing their specifications on an explicit and transparent foundation. The logistic function has been the default because it is automatically applied by the fsQCA software.

This paper has presented a key reason for motivating a more conscious choice of a membership function, namely the increase in empirical relevance as measured by coverage. In order to show how coverage can be increased while keeping functional form simple, the interactive effects between each one of four common baseline choices for the membership function and the anchoring of the crossover have been demonstrated by simulations. Irrespective of the crossover anchor, the standard logistic function has always been out-performed except under very narrow circumstances when positive contributions exactly cancel out negative ones. These situations seem highly unlikely to arise in practical research.

Viewing the choice of membership function from the perspective of coverage maximization opens up a set of questions future research should address. One avenue is the development of a procedure which identifies that membership function, or possibly set of functions, which maximizes coverage under a number of constraints. These must include pre-defined anchors. They may also include context-informed restrictions regarding concentration and dilation, functional simplicity, and a minimum level of set-theoretic consistency. The procedure need not be limited to the four options suggested herein, but could extend to whole classes of functions. This article can therefore only be a first awareness-raising step in that direction. But the set of questions does not end here.

Applications of fsQCA with a comparative focus are often interested in the location of specific cases rather than generalizations. These also include “non-cases” inconsistent with the set-theoretic relation that often attract attention. It has been demonstrated that different membership functions may not cause coverage to change while still substantially altering the location of individual cases within the set-theoretic relation between condition and outcome. Case-oriented studies have to remain alert to this fact and find ways to address inconsistencies that may result from the use of different membership functions.

One of the defining characteristics of QCA is the possibility for equifinality in explaining the outcome. In point of fact, the conjunction of causal combinations is the rule rather than the exception in applied research. However, for the sake of simplicity and focus, equifinality has been ignored in this paper. What effects the use of coverage in the determination of transformational rules produces in the context of more complex minimal formulas should thus also be examined.

Lastly, but perhaps the most desirable future development from the macro perspective on the continuing debate between proponents of “traditional” regression analysis and (fs)QCA would be a clear shift in the ground on which both camps have hitherto met. The vindication of QCA as a method and its juxtaposition with regression analysis should give way now to more fruitful advances in knowledge about possibilities to learn from the advantages of either tool kit of social enquiry.

References

Achen, Christopher H. 1990. What Does "Explained Variance" Explain?: Reply. *Political Analysis* 2 (1):173-184.

Banchoff, Thomas. 1999. German Identity and European Integration. *European Journal of International Relations* 5 (3):259-289.

Berg-Schlosser, Dirk, Gisèle de Meur, Benoît Rihoux, and Charles C. Ragin. 2009. Qualitative Comparative Analysis (QCA) as an Approach. In *Configurational Comparative Methods: Qualitative Comparative Analysis (QCA) and Related Techniques*, edited by B. Rihoux and C. C. Ragin. London: Sage Publications.

Bojadziev, George, and Maria Bojadziev. 2007. *Fuzzy Logic for Business, Finance, and Management*. 2nd ed, *Advances in Fuzzy Systems - Applications and Theory*. New Jersey: World Scientific.

Christmann, Anna. 2010. Damoklesschwert Referendum? Die indirekte Wirkung ausgebauter Volksrechte auf die Rechte religioser Minderheiten. *Swiss Political Science Review* 16 (1):1-41.

Clark, Terry D., Jennifer M. Larson, John N. Mordeson, Joshua D. Potter, and Mark J. Wierman. 2008. *Applying Fuzzy Mathematics to Formal Models in Comparative Politics*, *Studies in Fuzziness and Soft Computing,*. Berlin: Springer.

Davidson, Russell, and James G. MacKinnon. 1981. Several Tests for Model Specification in the Presence of Alternative Hypotheses. *Econometrica* 49 (3):781-793.

Freitag, Markus, and Raphaela Schlicht. 2009. Educational Federalism in Germany: Foundations of Social Inequality in Education. *Governance* 22 (1):47-72.

King, Gary. 1990. Stochastic Variation: A Comment on Lewis-Beck and Skalaban's "The R-Squared". *Political Analysis* 2 (1):185-200.

Klir, George J., Ute H. St. Clair, and Bo Yuan. 1997. *Fuzzy Set Theory: Foundations and Applications*. Upper Saddle River, NJ: Prentice Hall.

Koenig-Archibugi, Mathias. 2004. Explaining Government Preferences for Institutional Change in EU Foreign and Security Policy. *International Organization* 58 (1):137-174.

Kosko, Bart. 1993. *Fuzzy Thinking: The New Science of Fuzzy Logic*. New York: Hyperion.

Kvist, Jon. 2006. Diversity, Ideal Types and Fuzzy Sets in Comparative Welfare State Research. In *Innovative Comparative Methods for Policy Analysis*, edited by B. Rihoux and H. Grimm. New York: Springer Science+Business Media.

Lewis-Beck, Michael S., and Andrew Skalaban. 1990. The R-Squared: Some Straight Talk. *Political Analysis* 2 (1):153-171.

Marcussen, Martin, Thomas Risse, Daniela Engelmann-Martin, and Hans Joachim Knopf Klaus Roscher. 1999. Constructing Europe? The evolution of French, British and German nation state identities. *Journal of European Public Policy* 6 (4):614-633.

Marks, Gary, Liesbet Hooghe, and Arjan H. Schakel. 2008. Measuring Regional Authority. *Regional & Federal Studies* 18 (2):111-121.

Mizon, Grayham E., and Jean-Francois Richard. 1986. The Encompassing Principle and its Application to Testing Non-Nested Hypotheses. *Econometrica* 54 (3):657-678.

Nurmi, Hannu, and Janusz Kacprzyk. 2007. Fuzzy Sets in Political Science: An Overview. *New Mathematics and Natural Computation* 3 (3):281-299.

Pennings, Paul. 2003. Beyond Dichotomous Explanations: Explaining Constitutional Control of the Executive with Fuzzy-Sets. *European Journal of Political Research* 42 (4):541-567.

Ragin, Charles C. 2000. *Fuzzy-Set Social Science*. Chicago: University of Chicago Press.

———. 2006. Set Relations in Social Research: Evaluating Their Consistency and Coverage. *Political Analysis* 14 (3):291-310.

———. 2008. *Redesigning Social Inquiry: Fuzzy Sets and Beyond*. Chicago: University of Chicago Press.

———. 2009. Qualitative Comparative Analysis Using Fuzzy Sets (fsQCA). In *Configurational Comparative Methods: Qualitative Comparative Analysis (QCA) and Related Techniques*, edited by B. Rihoux and C. C. Ragin. London: Sage Publications.

Ragin, Charles C., Kriss A. Drass, and Sean Davey. 2006. *Fuzzy-Set/Qualitative Comparative Analysis 2.0*. Tucson: Department of Sociology, University of Arizona.

Ragin, Charles C., and Peer Fiss. 2008. Net Effects versus Configurations: An Empirical Demonstration. In *Redesigning Social Inquiry: Fuzzy Sets and Beyond*, edited by C. C. Ragin. Chicago: University of Chicago Press.

Ragin, Charles C., Sarah Ilene Strand, and Claude Rubinson. 2008. *User's Guide to Fuzzy-Set / Qualitative Comparative Analysis*. Tucson: University of Arizona.

Ramsey, J. B. 1969. Tests for Specification Errors in Classical Linear Least-Squares Regression Analysis. *Journal of the Royal Statistical Society. Series B (Methodological)* 31 (2):350-371.

Rihoux, Benoît, and Charles C. Ragin. 2009. Introduction. In *Configurational Comparative Methods: Qualitative Comparative Analysis (QCA) and Related Techniques*, edited by C. C. Ragin and B. Rihoux. London: SAGE.

———, eds. 2009. *Configurational Comparative Methods: Qualitative Comparative Analysis (QCA) and Related Techniques*. London: SAGE.

Schneider, Carsten Q., and Claudius Wagemann. 2007. *Qualitative Comparative Analysis und Fuzzy Sets: Ein Lehrbuch für Anwender und jene, die es werden wollen*. Opladen: Budrich.

Skaaning, Svend-Erik. 2007. Explaining Post-Communist Respect for Civil Liberty: A Multi-Methods Test. *Journal of Business Research* 60 (5):493-500.

Smithson, Michael. 1987. *Fuzzy Set Analysis for Behavioral and Social Sciences*. New York: Springer.

Smithson, Michael, and Jay Verkuilen. 2006. *Fuzzy Set Theory: Applications in the Social Sciences, Quantitative Applications in the Social Sciences*. London: SAGE.

Thiem, Alrik. 2010. Conditions of Intergovernmental Armaments Cooperation in Western Europe, 1996-2006. *European Political Science Review* 2 (3):forthcoming.

Verkuilen, Jay. 2005. Assigning Membership in a Fuzzy Set Analysis. *Sociological Methods Research* 33 (4):462-496.

Veugelers, J., and A. Magnan. 2005. Conditions of Far-Right Strength in Contemporary Western Europe: An Application of Kitschelt's Theory. *European Journal of Political Research* 44 (6):837-860.

Vis, Barbara. 2009. Governments and Unpopular Social Policy Reform: Biting the Bullet or Steering Clear? *European Journal of Political Research* 48 (1):31-57.

Zadeh, Lofti A. 1965. Fuzzy Sets. *Information and Control* 8 (3):338–353.